Chapter 15 Recursion

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

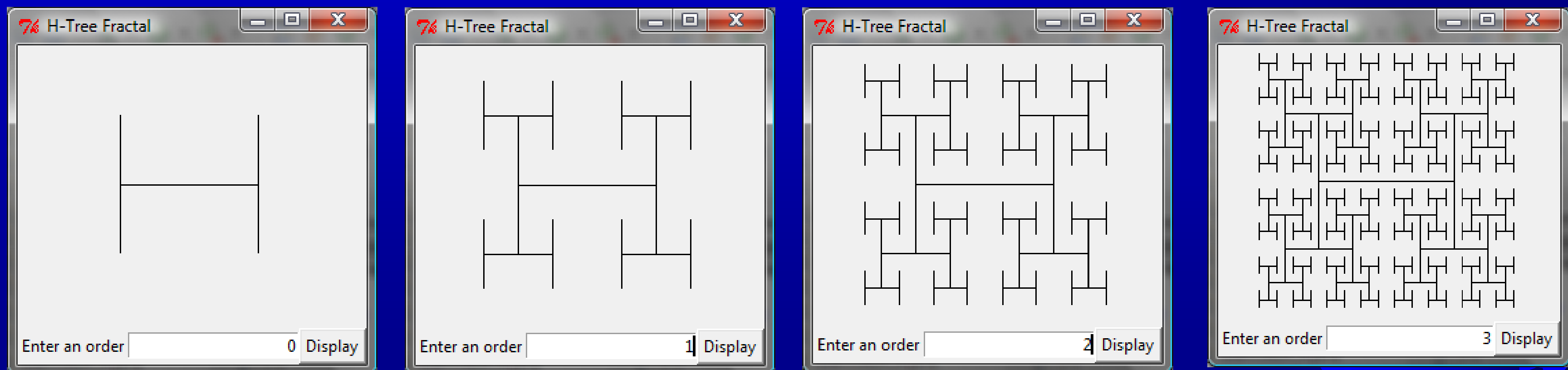
Motivations

Suppose you want to find all the files under a directory that contains a particular word. How do you solve this problem? There are several ways to solve this problem. An intuitive solution is to use recursion by searching the files in the subdirectories recursively.

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Motivations

The H-tree is used in VLSI design as a clock distribution network for routing timing signals to all parts of a chip with equal propagation delays. How do you write a program to display the H-tree? A good approach to solve this problem is to use recursion.



© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Objectives

- To describe what a recursive function is and the benefits of using recursion (§15.1).
- To develop recursive functions for recursive mathematical functions (§§15.2–15.3).
- To explain how recursive function calls are handled in a call stack (§§15.2–15.3).
- To use a helper function to derive a recursive function (§15.5).
- To solve selection sort using recursion (§15.5.1).
- To solve binary search using recursion (§15.5.2).
- To get the directory size using recursion (§15.6).
- To solve the Towers of Hanoi problem using recursion (§15.7).
- To draw fractals using recursion (§15.8).
- To solve the Eight Queens problem using recursion (§15.9).
- To discover the relationship and difference between recursion and iteration (§15.10).
- To know tail-recursive functions and why they are desirable (§15.11).

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

n! = n * (n-1)!

ComputeFactorial

Run

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

factorial(3)

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(3) = 3 * factorial(2)

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(3) = 3 * factorial(2)

= 3 * (2 * factorial(1))

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

factorial(3) = 3 * factorial(2)
= 3 * (2 * factorial(1))
= 3 * (2 * (1 * factorial(0)))

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(3) = 3 * factorial(2)

= 3 * (2 * factorial(1))

= 3 * (2 * (1 * factorial(0)))

= 3 * (2 * (1 * 1)))

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

factorial(3) = 3 * factorial(2)
= 3 * (2 * factorial(1))
= 3 * (2 * (1 * factorial(0)))
= 3 * (2 * (1 * 1)))
= 3 * (2 * 1)

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Computing Factorial

```
factorial(0) = 1;
factorial(n) = n*factorial(n-1);
```

factorial(3) = 3 * factorial(2)

= 3 * (2 * factorial(1))

= 3 * (2 * (1 * factorial(0)))

= 3 * (2 * (1 * 1)))

= 3 * (2 * 1)

= 3 * 2

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)
= 4 * 3 * factorial(2)
= 4 * 3 * (2 * factorial(1))
= 4 * 3 * (2 * (1 * factorial(0)))
= 4 * 3 * (2 * (1 * 1)))
= 4 * 3 * (2 * 1)
= 4 * 3 * 2
= 4 * 6
= 24

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Trace Recursive factorial

Executes factorial(4)

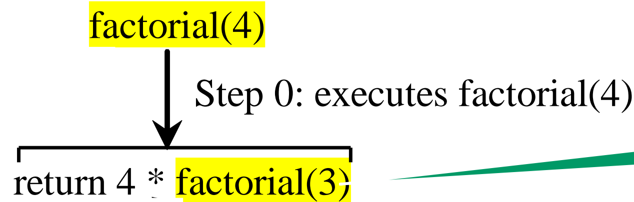
factorial(4)

Stack

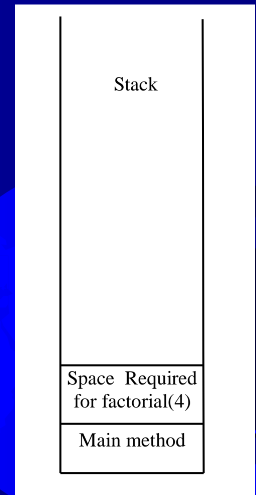
Main method

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Trace Recursive factorial



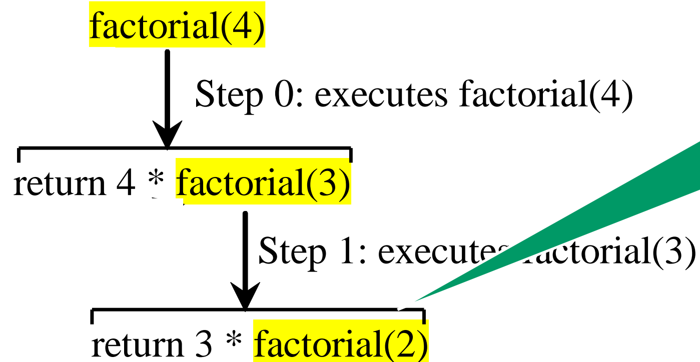
Executes factorial(3)



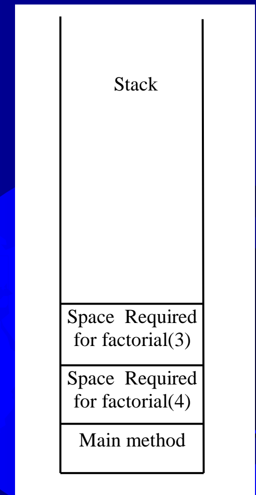
© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Trace Recursive factorial



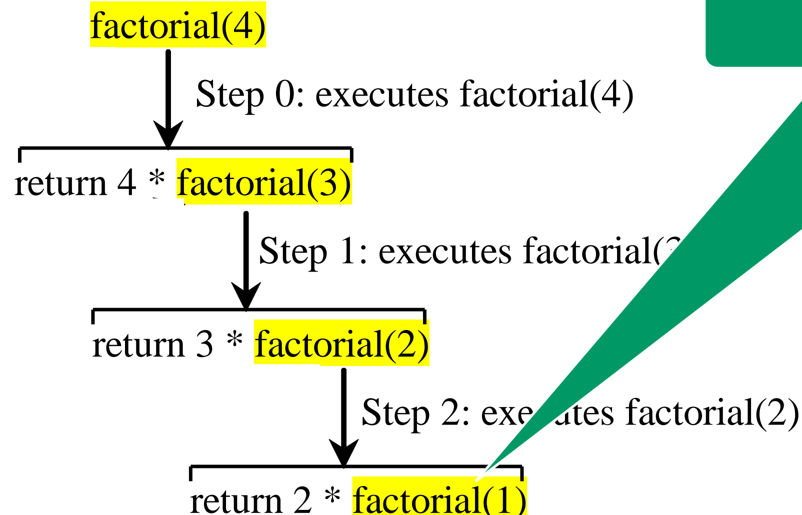
Executes factorial(2)

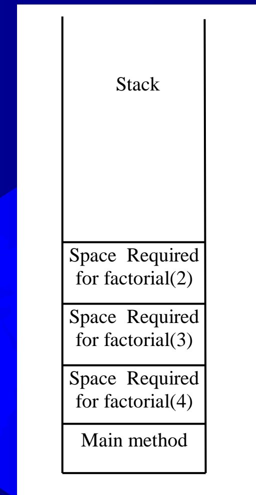


© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Trace Recursive factorial

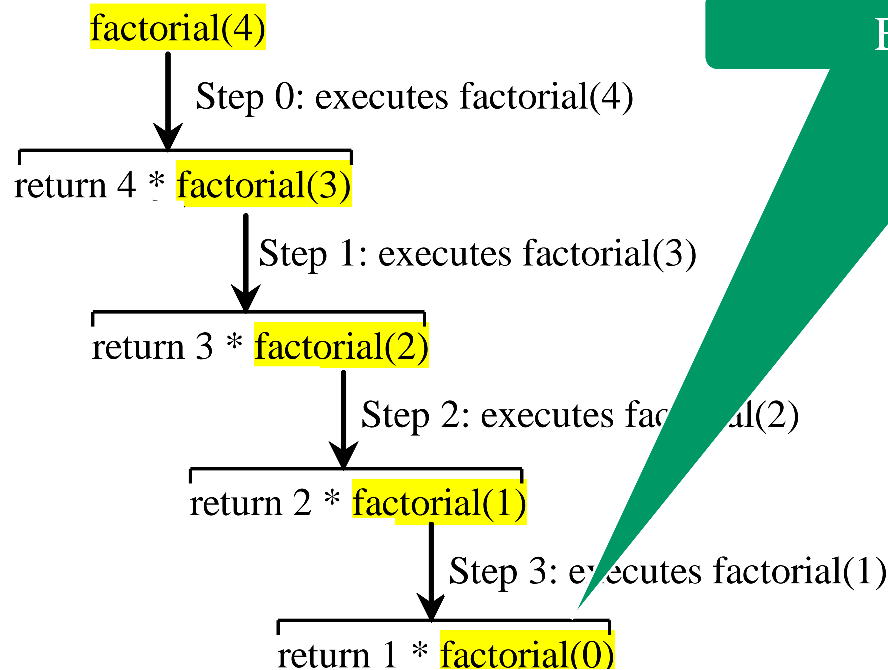




© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Trace Recursive factorial



Executes factorial(0)

Stack

Space Required for factorial(1)

Space Required for factorial(2)

Space Required for factorial(3)

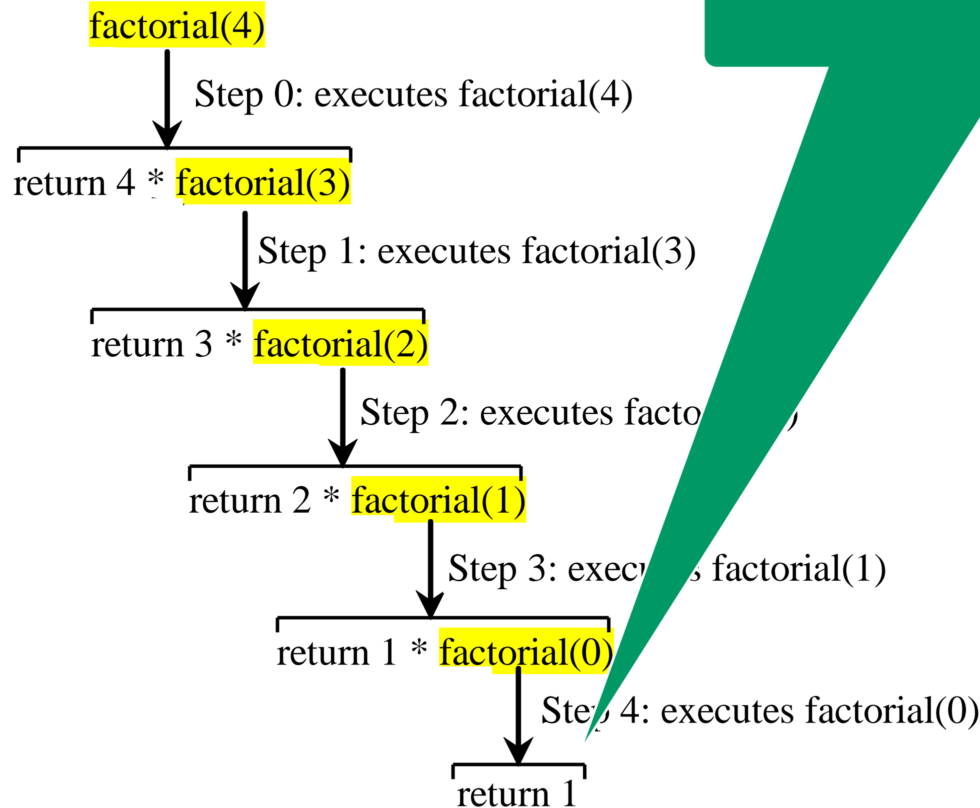
Space Required for factorial(4)

Main method

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Trace Recursive factorial



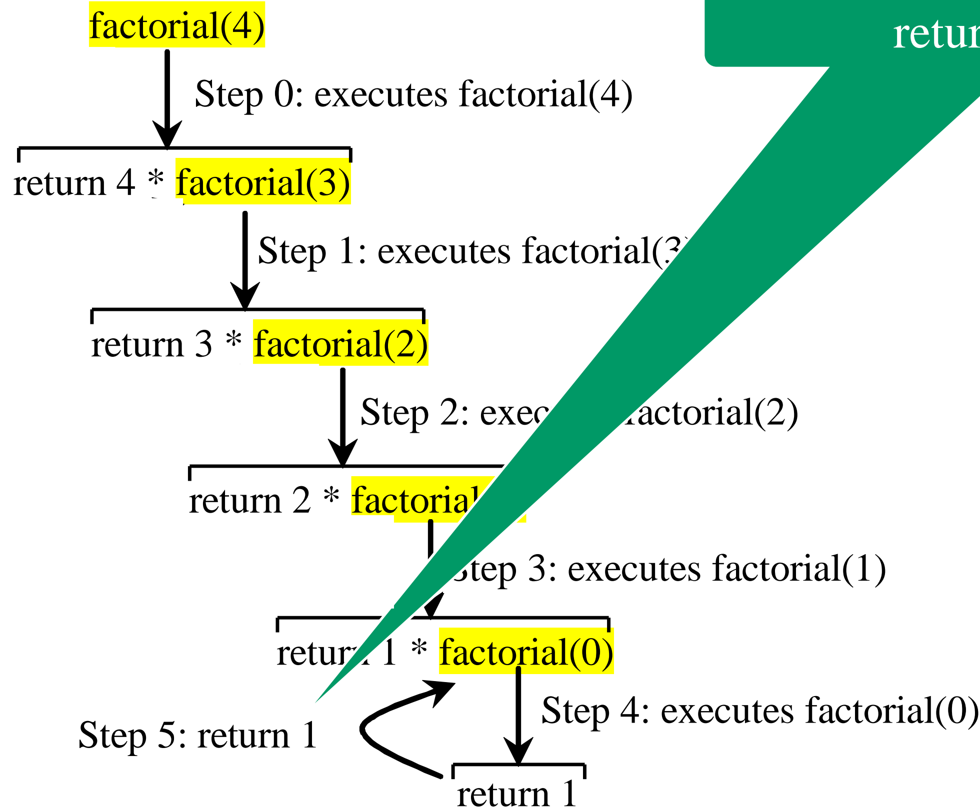
Stack

Stack
Space Required for factorial(0)
Space Required for factorial(1)
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Trace Recursive factorial

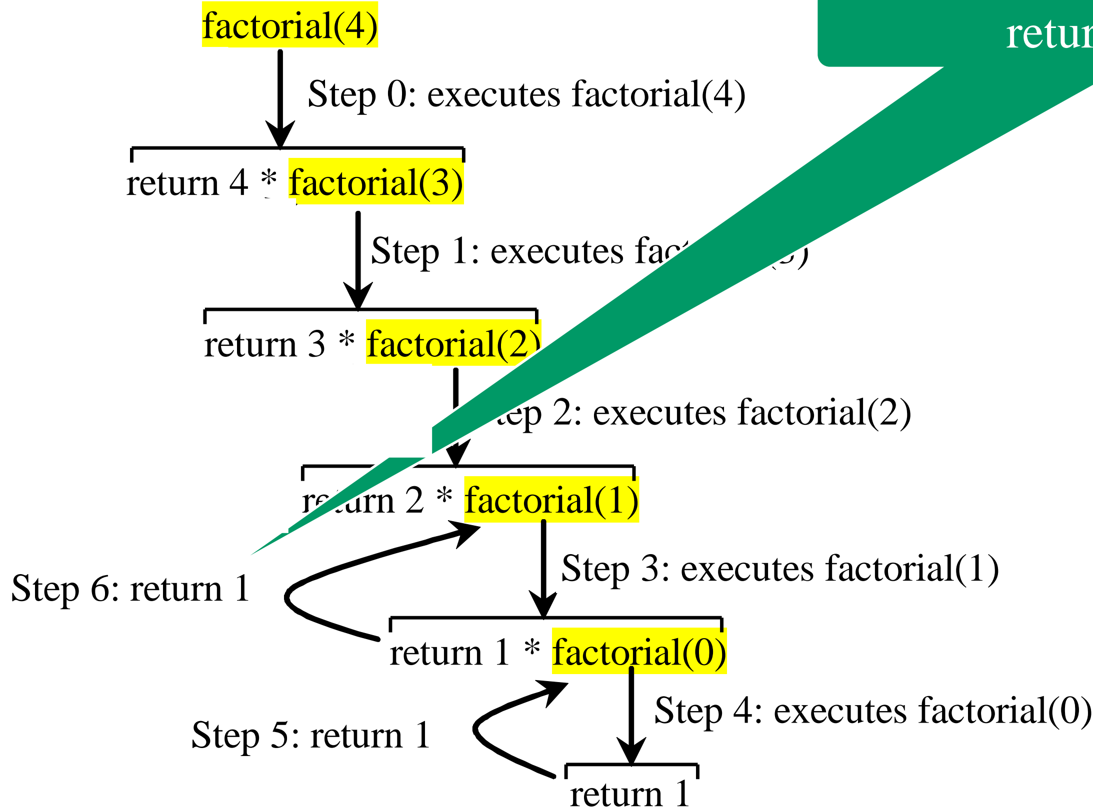


Stack
Space Required for factorial(1)
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Trace Recursive factorial

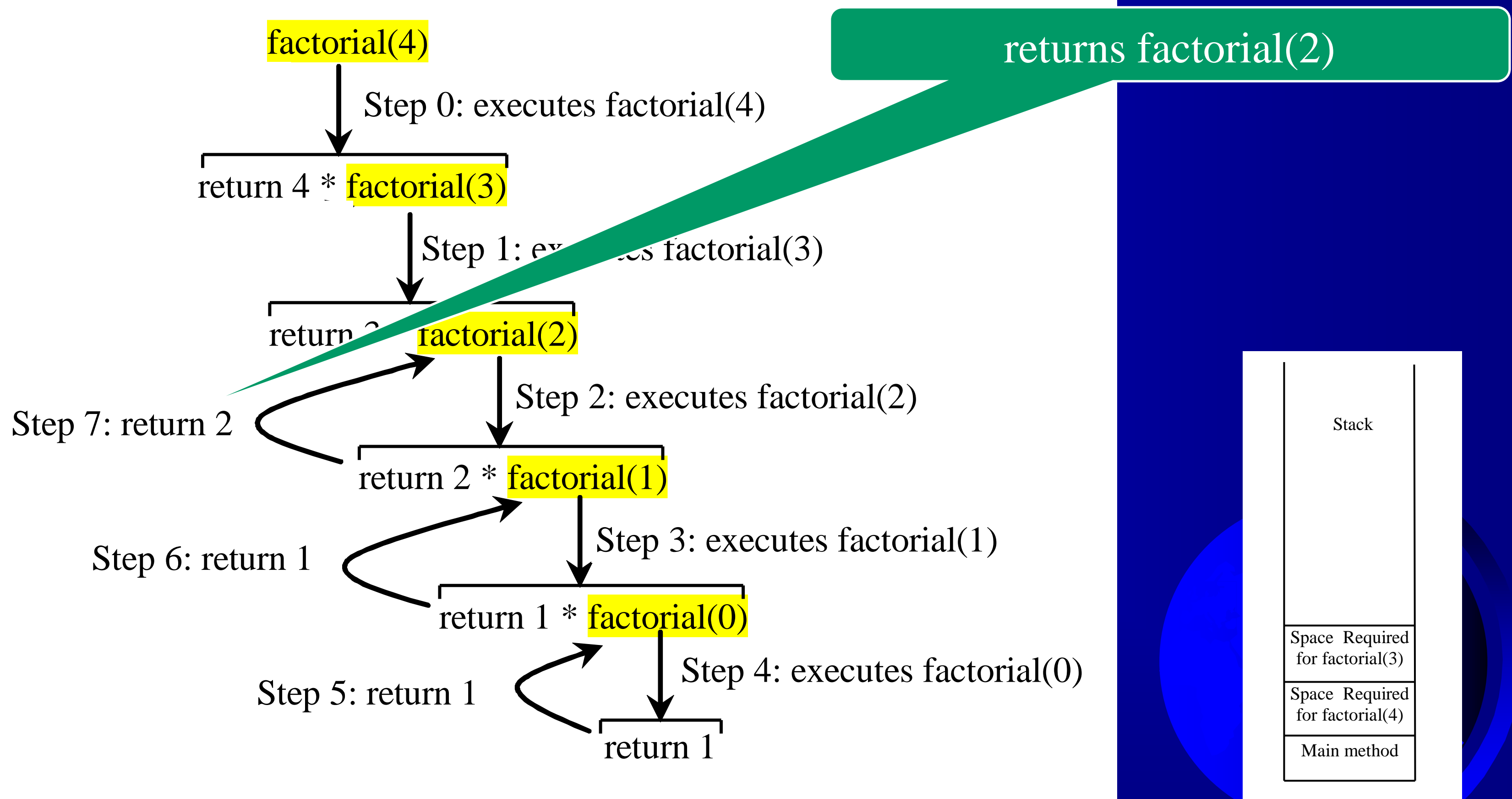


Stack
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

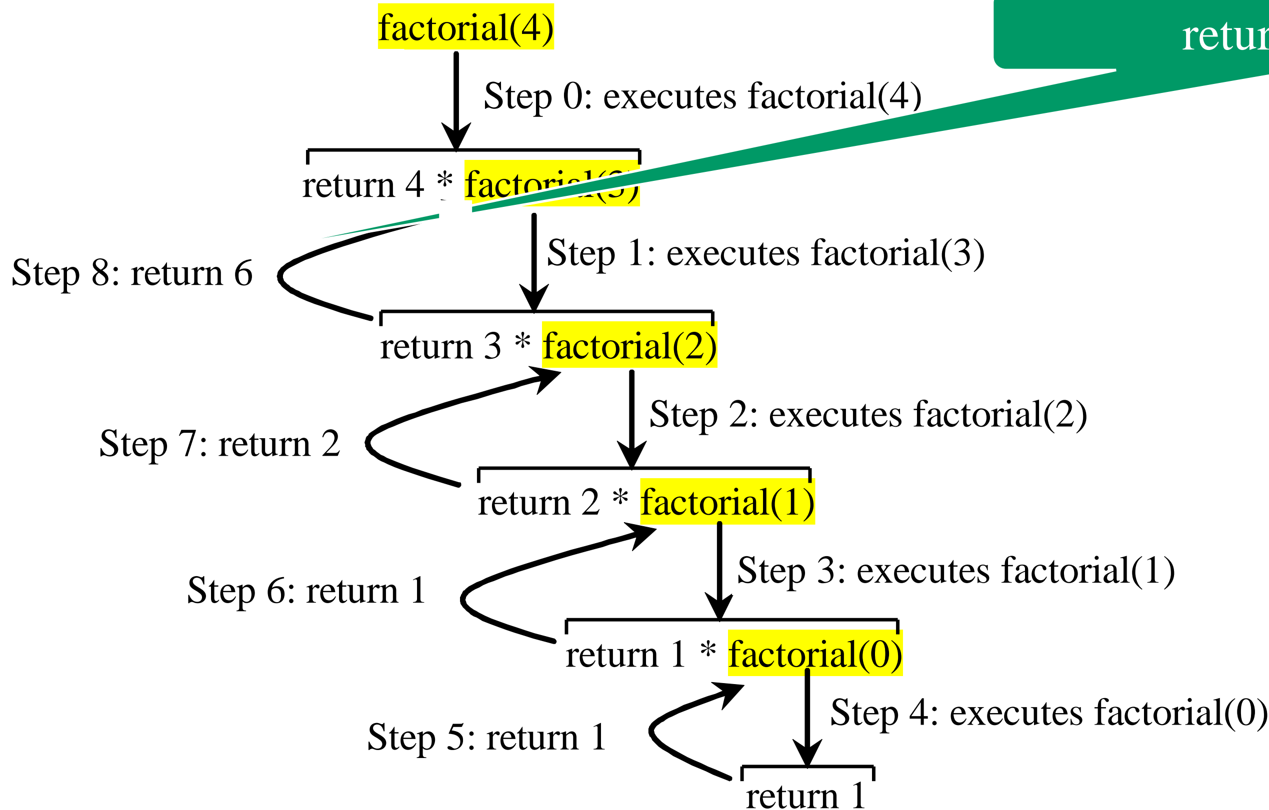
Trace Recursive factorial



© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Trace Recursive factorial



returns factorial(3)

Stack

Space Required for factorial(4)

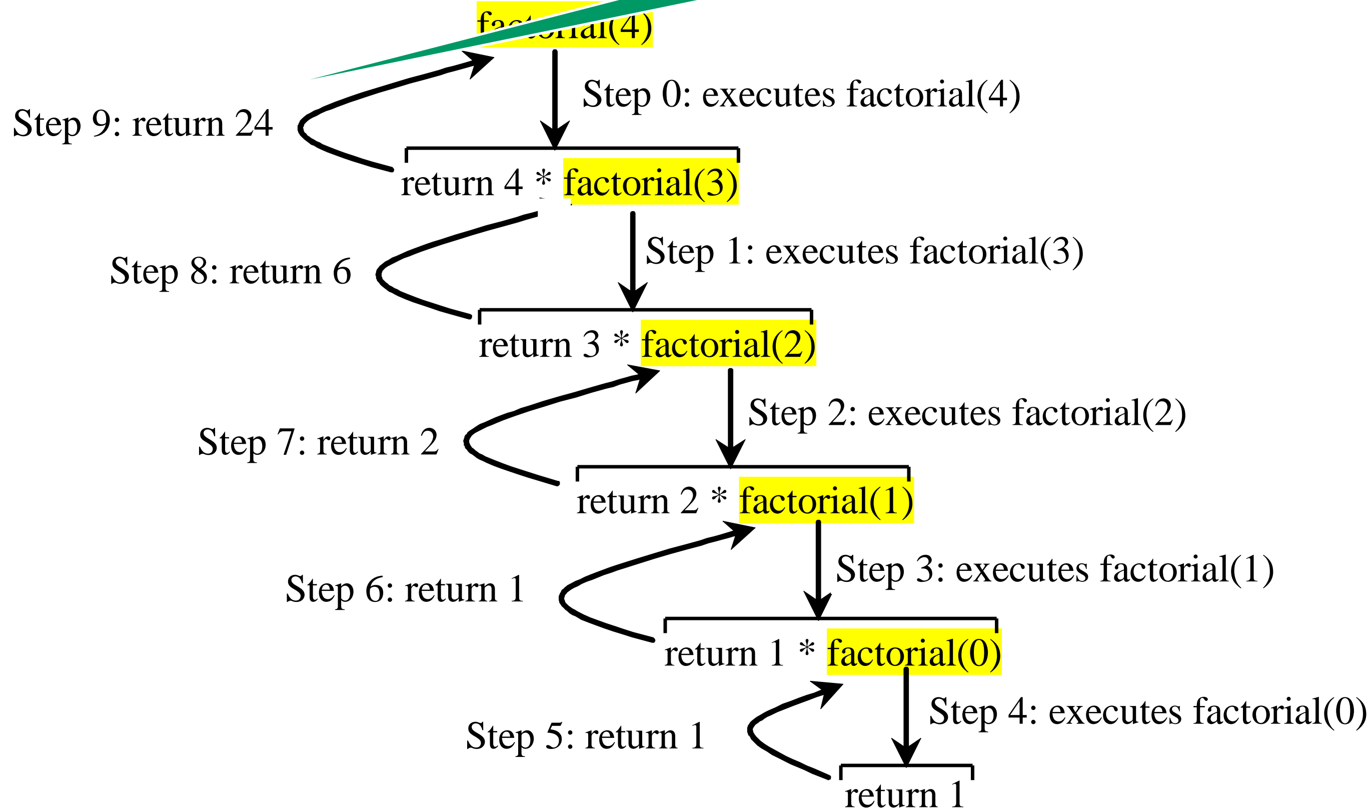
Main method

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

animation

Trace Recursive factorial

returns factorial(4)

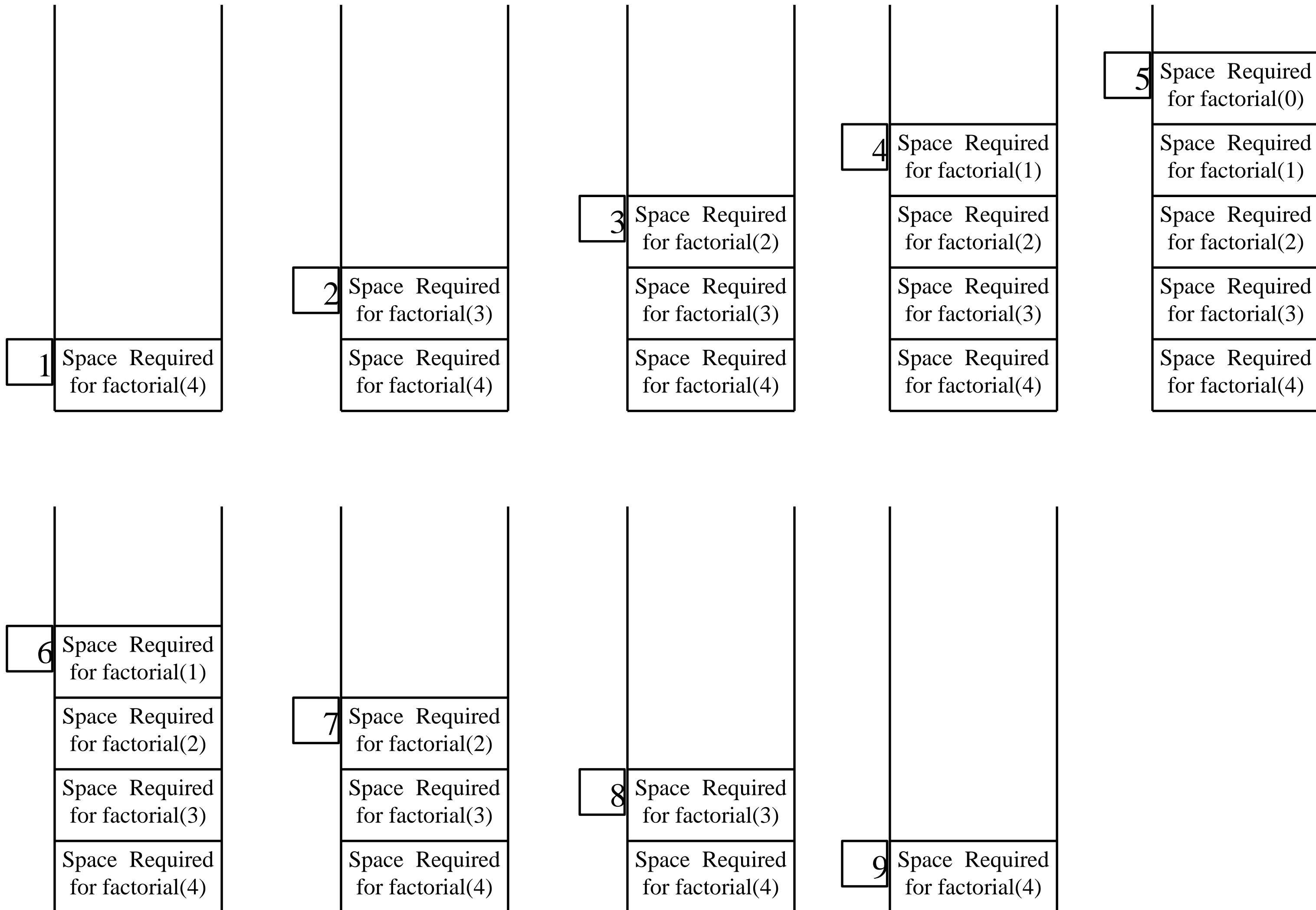


Stack

Main method

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

factorial(4) Stack Trace



© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Other Examples

$f(0) = 0;$

$f(n) = n + f(n-1);$

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Fibonacci Numbers

```
Fibonacci series: 0 1 1 2 3 5 8 13 21 34 55 89…
          indices: 0 1 2 3 4 5 6 7  8  9  10 11
```

fib(0) = 0;

fib(1) = 1;

fib(index) = fib(index -1) + fib(index -2); index >=2

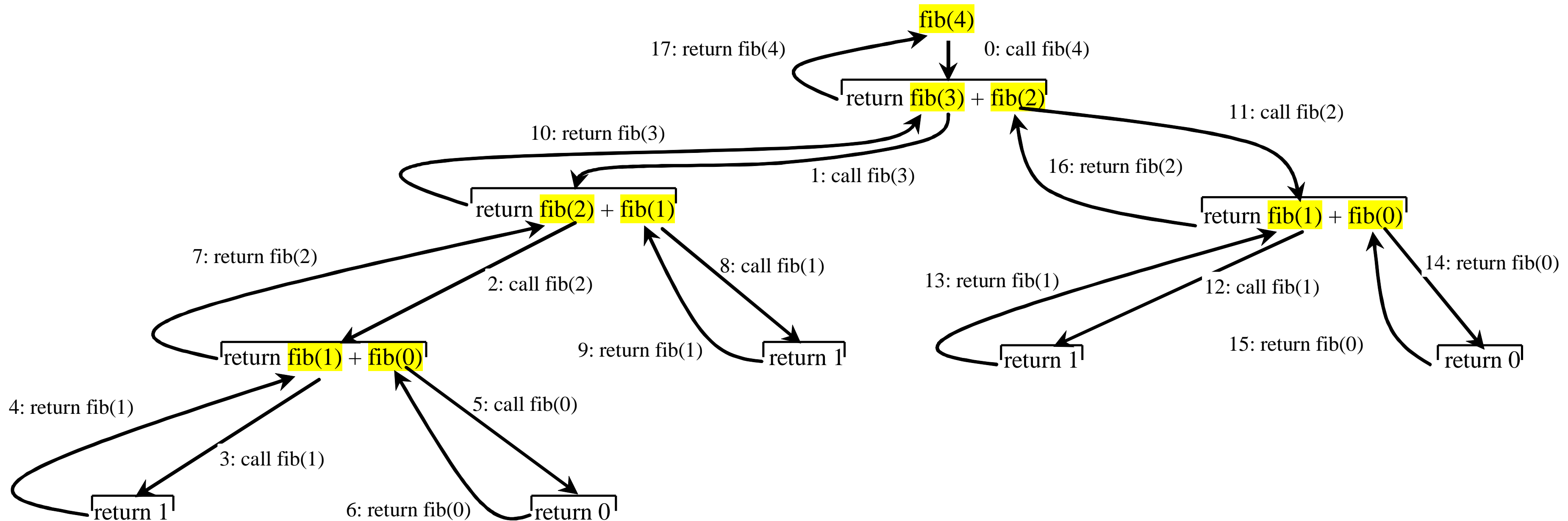
fib(3) = fib(2) + fib(1) = (fib(1) + fib(0)) + fib(1) = (1 + 0) +fib(1) = 1 + fib(1) = 1 + 1 = 2

ComputeFibonacci

Run

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Fibonnaci Numbers, cont.



© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Characteristics of Recursion

All recursive methods have the following characteristics:

- One or more base cases (the simplest case) are used to stop recursion.
- Every recursive call reduces the original problem, bringing it increasingly closer to a base case until it becomes that case.

In general, to solve a problem using recursion, you break it into subproblems. If a subproblem resembles the original problem, you can apply the same approach to solve the subproblem recursively. This subproblem is almost the same as the original problem in nature with a smaller size.

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Problem Solving Using Recursion

Let us consider a simple problem of printing a message for n times. You can break the problem into two subproblems: one is to print the message one time and the other is to print the message for n-1 times. The second problem is the same as the original problem with a smaller size. The base case for the problem is n==0. You can solve this problem using recursion as follows:

```
def nPrintln(message, times):
    if times >= 1:
        print(message)
        nPrintln(message, times - 1)
    # The base case is times == 0
```

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Think Recursively

Many of the problems presented in the early chapters can be solved using recursion if you *think recursively*. For example, the palindrome problem in Listing 8.1 can be solved recursively as follows:

```
def isPalindrome(s):
    if len(s) <= 1: # Base case
        return True
    elif s[0] != s[len(s) - 1]: # Base case
        return False
    else:
        return isPalindrome(s[1 : len(s) - 1])
```

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Recursive Helper Methods

The preceding recursive isPalindrome method is not efficient, because it creates a new string for every recursive call. To avoid creating new strings, use a helper method:

```
def isPalindrome(s):
    return isPalindromeHelper(s, 0, len(s) - 1)

def isPalindromeHelper(s, low, high):
    if high <= low: # Base case
        return True
    elif s[low] != s[high]: # Base case
        return False
    else:
        return isPalindromeHelper(s, low + 1, high - 1)
```

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Recursive Selection Sort

1. Find the smallest number in the list and swaps it with the first number.
2. Ignore the first number and sort the remaining smaller list recursively.

RecursiveSelectionSort

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Recursive Binary Search

1. Case 1: If the key is less than the middle element, recursively search the key in the first half of the array.
2. Case 2: If the key is equal to the middle element, the search ends with a match.
3. Case 3: If the key is greater than the middle element, recursively search the key in the second half of the array.

RecursiveBinarySearch

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

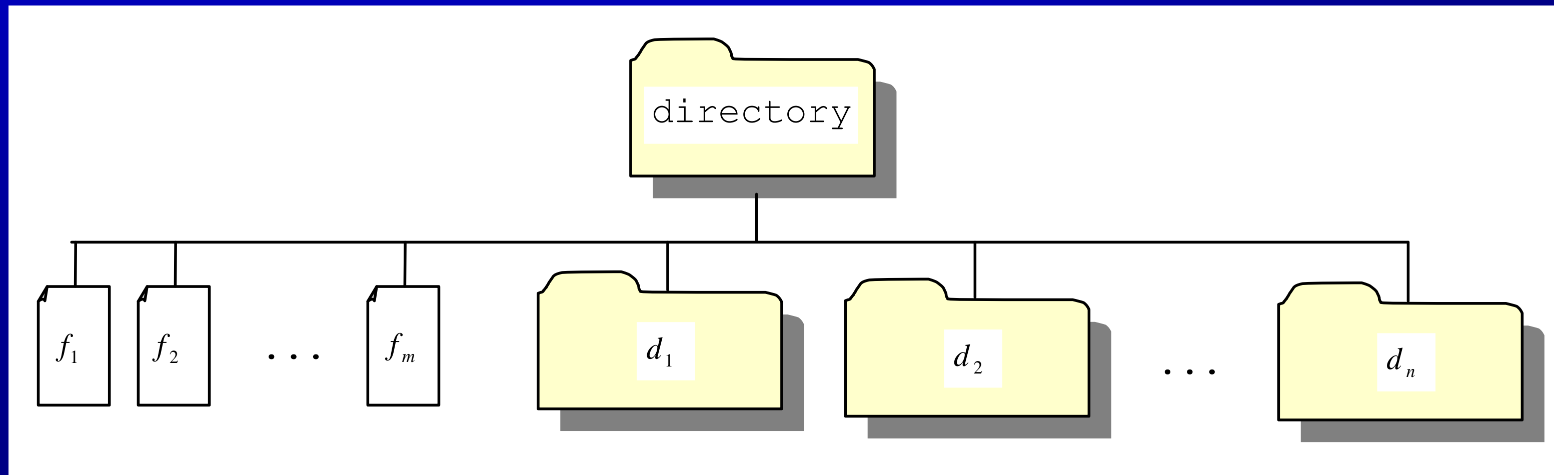
Recursive Implementation

```
def recursiveBinarySearch(list, key):
    low = 0
    high = len(list) - 1
    return recursiveBinarySearchHelper(list, key, low, high)

def recursiveBinarySearchHelper(list, key, low, high):
    if low > high:  # The list has been exhausted without a match
        return -low - 1

    mid = (low + high) // 2
    if key < list[mid]:
        return recursiveBinarySearchHelper(list, key, low, mid - 1)
    elif key == list[mid]:
        return mid
    else:
        return recursiveBinarySearchHelper(list, key, mid + 1, high)

def main():
    list = [3, 5, 6, 8, 9, 12, 34, 36]
    print(recursiveBinarySearch(list, 3))
    print(recursiveBinarySearch(list, 4))

main()
```

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Directory Size

The preceding examples can easily be solved without using recursion. This section presents a problem that is difficult to solve without using recursion. The problem is to find the size of a directory. The size of a directory is the sum of the sizes of all files in the directory. A directory may contain subdirectories. Suppose a directory contains files , , ..., , and subdirectories , , ..., , as shown below.



© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Directory Size

The size of the directory can be defined recursively as follows:

$$size(d) = size(f_1) + size(f_2) + ... + size(f_m) + size(d_1) + size(d_2) + ... + size(d_n)$$

DirectorySize

Run

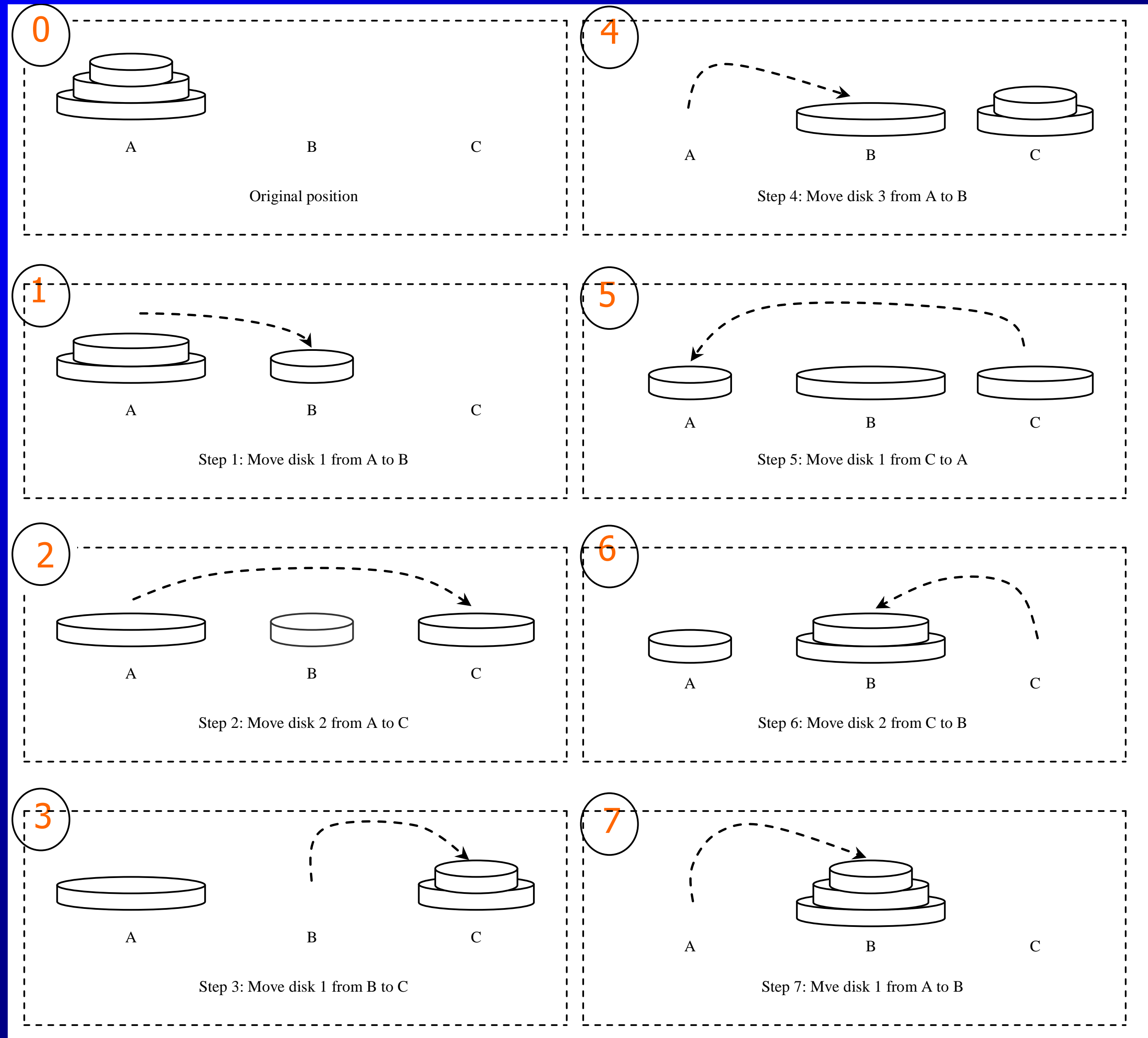
© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Towers of Hanoi

- There are n disks labeled 1, 2, 3, . . ., n, and three towers labeled A, B, and C.
- No disk can be on top of a smaller disk at any time.
- All the disks are initially placed on tower A.
- Only one disk can be moved at a time, and it must be the top disk on the tower.

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

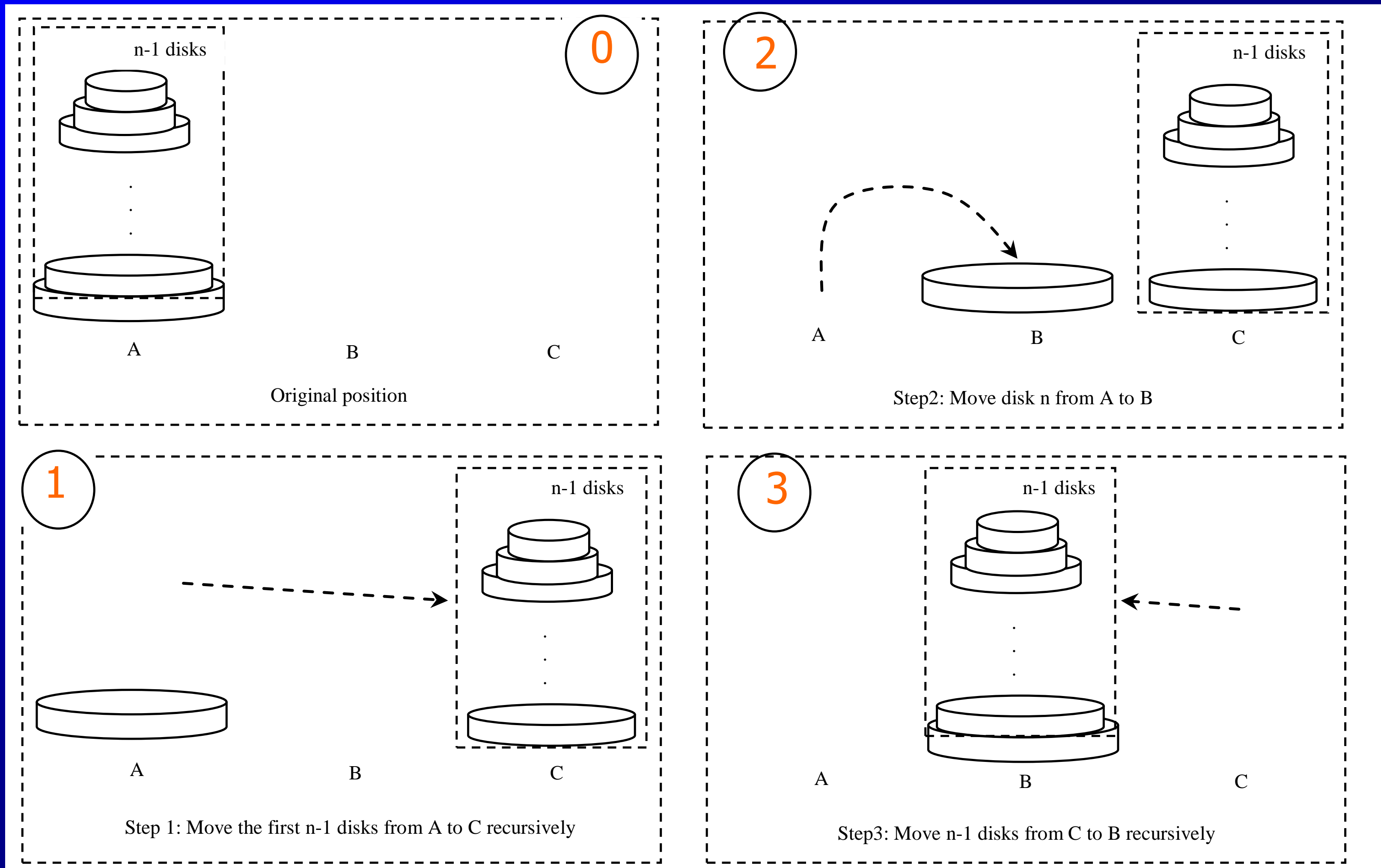
Towers of Hanoi, cont.



© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Solution to Towers of Hanoi

The Towers of Hanoi problem can be decomposed into three subproblems.



© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Solution to Towers of Hanoi

- Move the first n - 1 disks from A to C with the assistance of tower B.
- Move disk n from A to B.
- Move n - 1 disks from C to B with the assistance of tower A.

TowersOfHanoi

Run

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

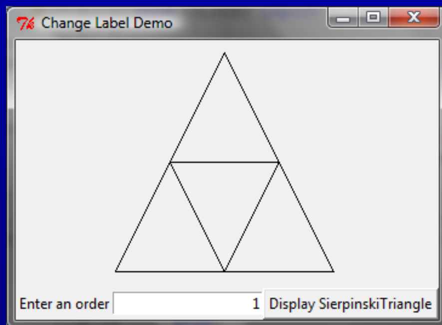
Fractals?

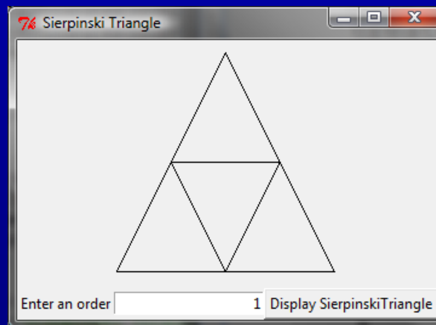
A fractal is a geometrical figure just like triangles, circles, and rectangles, but fractals can be divided into parts, each of which is a reduced-size copy of the whole. There are many interesting examples of fractals. This section introduces a simple fractal, called *Sierpinski triangle*, named after a famous Polish mathematician.

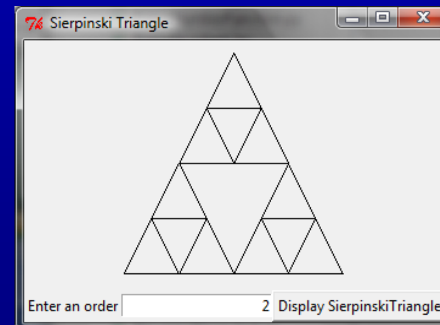
© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

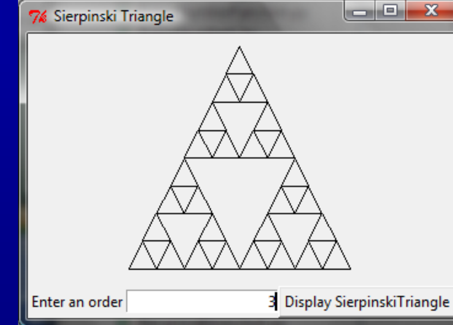
Sierpinski Triangle

1. It begins with an equilateral triangle, which is considered to be the Sierpinski fractal of order (or level) 0, as shown in Figure (a).
2. Connect the midpoints of the sides of the triangle of order 0 to create a Sierpinski triangle of order 1, as shown in Figure (b).
3. Leave the center triangle intact. Connect the midpoints of the sides of the three other triangles to create a Sierpinski of order 2, as shown in Figure (c).
4. You can repeat the same process recursively to create a Sierpinski triangle of order 3, 4, ..., and so on, as shown in Figure (d).



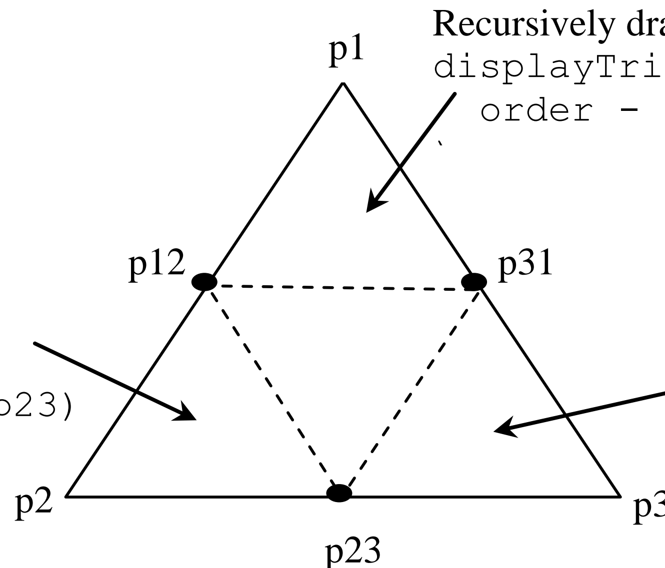






© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Sierpinski Triangle Solution



Recursively draw the small Sierpinski triangle

```
displayTriangles(
  order - 1, p1, p12, p31)
```

Recursively draw the small Sierpinski triangle

```
displayTriangles(
  order - 1, p12, p2, p23)
```

Recursively draw the small Sierpinski triangle

```
displayTriangles(
  order - 1, p31, p23, p3)
```

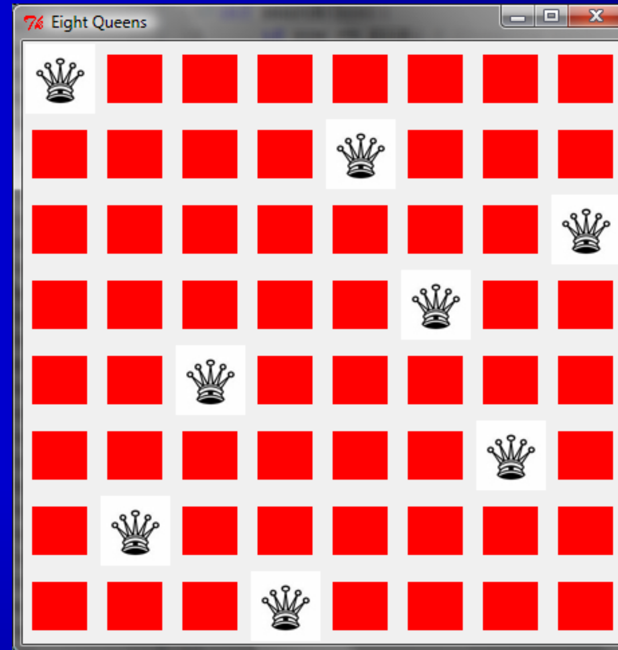
SierpinskiTriangle

Run

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Eight Queens

queens[0]	0
queens[1]	4
queens[2]	7
queens[3]	5
queens[4]	2
queens[5]	6
queens[6]	1
queens[7]	3



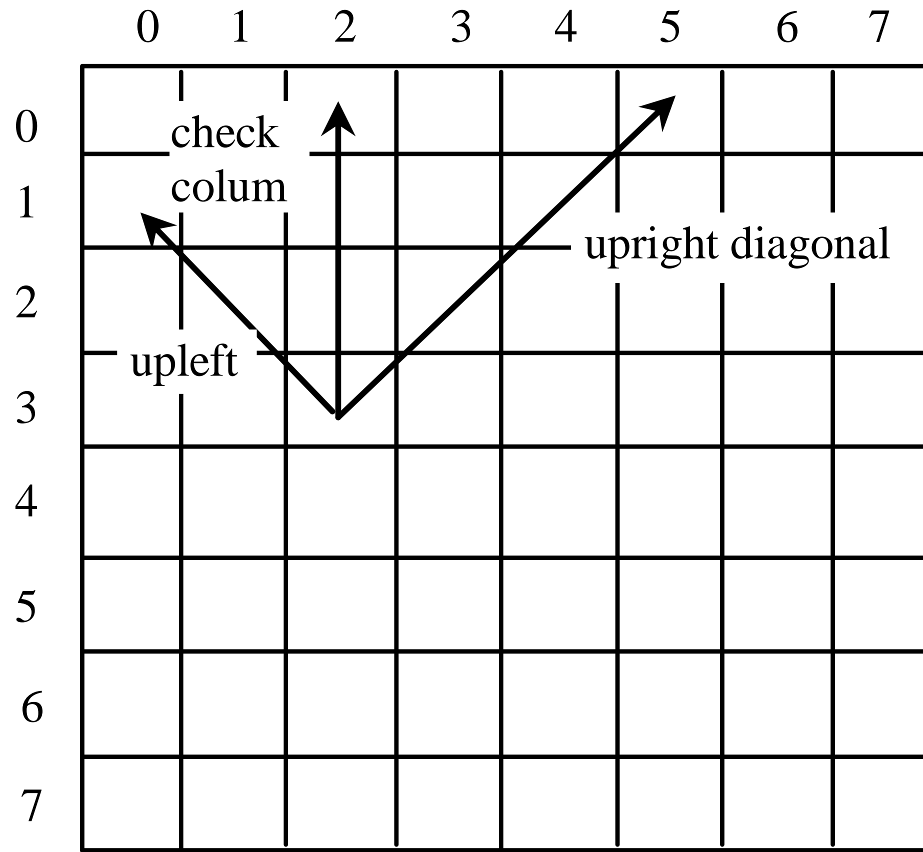
- The Eight Queens problem is to find a solution to place a queen in each row on a chessboard so that no two queens can attack each other.
- The program arranges eight queens on a chessboard. There can only be one queen in each row, and the queens must be positioned such that no two queens can take the other.

EightQueens

Run

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Eight Queens



© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Recursion vs. Iteration

Recursion is an alternative form of program control. It is essentially repetition without a loop.

Recursion bears substantial overhead. Each time the program calls a method, the system must assign space for all of the method's local variables and parameters. This can consume considerable memory and requires extra time to manage the additional space.

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Advantages of Using Recursion

Recursion is good for solving the problems that are inherently recursive.

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.

Tail Recursion

A recursive method is said to be *tail recursive* if there are no pending operations to be performed on return from a recursive call.

Non-tail recursive — ComputeFactorial

Tail recursive — ComputeFactorialTailRecursion

© Copyright 2012 by Pearson Education, Inc. All Rights Reserved.